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The Fracture Threshold for an Adhesive Interlayer

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Synopsis

Complementing an earlier paper which utilized an energy balance criterion for a continuum mechanics analysis of adhesive failure in a pressurized blister at the interface of an elastic material and a rigid substrate, the analysis is extended to include an additional elastic interlayer between them. An infinite lateral-length elastic plate strip bonded through a Winkler elastic foundation to a rigid substrate is assumed, in which the plate is separated from the adhesive layer by internal pressure. It is found that the important design parameters are the tensile modulus-to-thickness ratio of the adhesive layer and the adhesive fracture energy of separation of the respective materials. The results provide a basis for investigating changes in the chemical microstructure of the adhesive.

INTRODUCTION

One of the more important aspects of the adhesive debonding problem is the quantitative importance of the thickness and mechanical properties of the adhesive layer which lies between two adhering surfaces. Practical experience has indicated a significance, usually tending toward thin adhesive layers, but it appears that limited quantitative analysis of this phenomenon has been furnished. Building upon the original specimen configuration suggested by Dannenberg[1] and amplified in certain analytical respects by Malyshev and Salganik,[2] the author has proposed a modified "pressurized blister test" to treat the case of a soft elastomeric material cast, and cured, upon a relatively rigid substrate.[3] In essence, a pressure inlet hole is drilled through the underside of the substrate and the applied pressure required to lift the layer off the surface, forming a blister and extending its radius, is measured.

A Griffith-type criterion for adhesive fracture is adopted, wherein the change in internal strain energy stored in the layer with increasing blister size or radius is equated to the associated increment in the amount of energy absorbed in creating the newly fractured surface area. For the particular case of a linearly elastic plate-like disk cast against a rigid surface, it was found for example that the pressure, p_{cr}, above which the blister would increase in size, was

$$p_{cr} = \left[\frac{32}{3(1 - \nu^2)}\left(\frac{h}{a}\right)^3\right]^{1/2}\sqrt{\frac{E\gamma_a}{a}} \tag{1}$$

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where, in calculating the strain energy stored, linear plate theory† was used and h/a is the plate thickness-to-plate radius ratio, E and ν are the tensile modulus and Poisson's ratio of the material, and γ_a (in.-lb/in.^{2}) is the specific adhesive fracture energy of the surface defined through the surface energy S, such that $S = \pi a^2 \gamma_a$. Tests were run from which γ_a was deduced from measurements of the critical pressure at a corresponding radius for known material properties. As a matter of fact, for a particular urethane rubber against glass, γ_a was found to be 1.4 in.-lb/in.^{2} Assuming the same materials and surface preparation in a different specimen configuration, it was shown how this now known value of γ_a could be used to predict, a priori, the adhesive debonding for a different applied loading.

In many cases however, one material is not cast and cured against the second material, but rather it is bonded to it by an intermediate, usually thin, layer of a different material. The aforementioned analysis does not specifically treat this important class of problems, and it is proposed to illustrate the modifications which can be adopted to do so.

THE ELASTIC ADHESIVE INTERFACE

As a model of the phenomenon involved, consider instead of a circular blister, an elastic plate strip of material properties E and ν, and thickness h, infinite in the lateral y-direction (plane strain) and appearing in cross section essentially as a beam (Fig. 1). The plate specimen is mounted upon an interface elastic adhesive layer of material properties E' and ν' and thickness h'. The substrate underneath the adhesive layer is considered infinitely rigid, as before. From the standpoint of linear plate theory, the

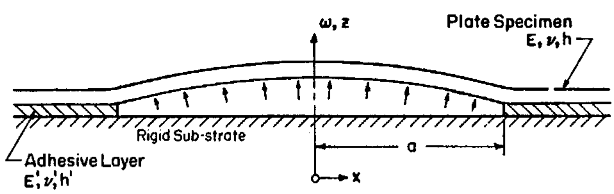

Fig. 1. Cross section of the configuration.

problem can be formulated by assuming an equivalent Winkler elastic foundation[5] modulus k, which leads then to a simple analysis of the classic beam supported by an elastic foundation. (It should be noted that the essential assumption introduced by Winkler[5] allows for vertical motion and dilatation stress only. Shear effects in the foundation can be incorporated, however.[6]) The deflection of the central portion of the strip, $|x| < a$, loaded by pressure only, is then determined from standard beam theory which, for the outer portions, $|x| > a$ (that supported by the elastic foundation), is found from the Winkler equation. The deflection, slope,

† Jones and Selto have recently extended the analysis to include the case of a thin (nonlinear) membrane layer.[4]

moment, and shear are matched at $|x| = a$ to complete the solution. The essential details follow.

The Inner Solution, $|x| < a$. For this region the governing differential equation is

$$D(d^4w/dx^4) = p \tag{2}$$

with the solution

$$Dw(x) = (px^4/24) + (C_2x^2/2) + C_0 \tag{3}$$

where the flexural rigidity is $D = Eh^3/[12(1 - \nu^2)]$.

The Outer Solution, $|x| > a$. In this region, the Winkler equation is

$$D(d^4w/dx^4) + kw = 0 \tag{4}$$

with the solution

$$w(x) = (C_3 \cos \lambda x + C_4 \sin \lambda x) \exp(-\lambda x) \tag{5}$$

where that part of the solution corresponding to $\exp(+\lambda x)$ has been neglected due to the assumed lateral infinite extent of the adhesive layer. The constant λ is defined by

$$k \equiv 4D\lambda^4. \tag{6a}$$

It is also useful to define for later use the dimensionless parameters representing the (damped) wavelength of the deformation in the foundation,

$$\mu \equiv \lambda a. \tag{6b}$$

The constants C_0, C_2, C_3, and C_4 are determined by requiring that the deflection, slope, moment, and shear—$w(a)$, $dw(a)/dx$, $D(d^2w/dx^2)$, and $D(d^3w/dx^3)$, respectively, for the inner and outer solutions match at $x = a$. An application of these conditions leads to

$$C_0 = pa^4\left[\frac{1}{24} + \frac{2\mu^3 + 5\mu^2 + 6\mu + 3}{12\mu^3(1 + \mu)}\right] \tag{7}$$

$$C_2 = -pa^2\left[\frac{1}{6} + \frac{2\mu + 3}{6\mu(1 + \mu)}\right] \tag{8}$$

$$C_3 = \frac{pa^4}{D \exp(-\lambda a)}\left[\frac{(2\mu^2 + 6\mu + 3) \cos \mu + (2\mu^2 - 3) \sin \mu}{12\mu^3(1 + \mu)}\right] \tag{9}$$

$$C_4 = \frac{pa^4}{D \exp(-\lambda a)}\left[\frac{(2\mu^2 + 6\mu + 3) \sin \mu - (2\mu^2 - 3) \cos \mu}{12\mu^3(1 + \mu)}\right] \tag{10}$$

ENERGY BALANCE

With the constants now determined, the strain energy per unit lateral length stored in the various parts of the specimen can be calculated. Using Clapeyron's theorem,[7] one finds that the energy in the plate is

$$U_1 = \frac{1}{2}\int_{-a}^{a} pw(x)\, dx, \tag{11}$$

whereupon using eq. (3), along with eqs. (7) and (8),

$$U = \frac{p^2a^5}{45D}\left\{1 + \frac{5}{4}\frac{4\mu^3 + 12\mu^2 + 18\mu + 9}{\mu^3(1 + \mu)}\right\} \tag{12}$$

where the first term in the brackets may be identified as the elementary solution contributed by the central portion of the plate strip alone with clamped edges at $|x| = a$, i.e., $\mu \to \infty$.

It is necessary next to calculate the increment in adhesive energy per unit lateral length to create new surface at the ends of the strip S. It is assumed that the debonding will take place along the interface between the plate and the adhesive layer and that the specific adhesive energy associated with the debonding is γ_a. Thus one has

$$S = 2a\gamma_a \tag{13}$$

The energy balance at criticality, $\partial U/\partial a = \partial S/\partial a$, upon noting (6b), leads easily to

$$p_{cr}^2 = \frac{\frac{3}{2}\left(\frac{h}{a}\right)^3\left(\frac{E\gamma_a}{a}\right)}{1 + \frac{16\mu^4 + 56\mu^3 + 84\mu^2 + 63\mu + 18}{4\mu^3(1 + \mu)^2}} \tag{14}$$

where the numerator is the clamped-clamped strip solution, i.e., $\mu \to \infty$, given earlier (Williams,[3] Table I, formula 4). The second term in the denominator therefore represents the effect and presence of an elastic foundation, within the Winkler approximation, and is the desired correction factor for a finite-thickness adhesive bonding layer.

It is useful to note that in many cases the parameter $\mu = \lambda a$ will be sufficiently large that the correction factor may be adequately represented by

$$p_{cr} \simeq \frac{3}{2}\left(\frac{h}{a}\right)^3 \frac{E\gamma_a}{a}\left[1 - \frac{4}{\mu} + \dots\right] \qquad ;\mu \to \infty \tag{15}$$

$$= \frac{3}{2}\left(\frac{h}{a}\right)^3 \frac{E\gamma a}{a}\left[1 - 4\sqrt[4]{\frac{1}{3}\left(\frac{h}{a}\right)^3\frac{E}{ka}} + \dots\right] ;\mu \to \infty \tag{16}$$

Also, the foundation modulus k may be estimated from the characteristics of the adhesive layer. Assuming for the thin layer that its lateral strain is inhibited ($\epsilon_x = \epsilon_y = 0$) and that only normal strain is permitted, which is consistent with the Winkler hypothesis, one finds that the foundation constant relating applied normal pressure and deflection, w, is*

$$k = \frac{1 - \nu'}{(1 - 2\nu')(1 + \nu')}\frac{E'}{h'} \tag{17}$$

* For a beam, rather than a plate strip, $k = [E'/h']/[1 - \nu'^2]$.

which permits a direct evaluation of $\mu = \lambda a$, viz.,

$$\frac{1}{\mu} = \frac{h}{a}\sqrt[4]{\frac{(1+\nu')(1-2\nu')}{3(1-\nu')(1-\nu^2)}\cdot\frac{E/h}{E'/h'}}. \tag{18}$$

For the case when the foundation is "stiff," $k \sim \mu^{1/4} \rightarrow \infty$,

$$p_{cr}^2(\mu) \simeq \simeq p_{cr}^2(\infty)\left[1 - 4\frac{h}{a}\cdot\sqrt[4]{\frac{(1+\nu')(1-2\nu')}{3(1-\nu')(1-\nu^2)}\frac{E/h}{E'/h'}}\right] \tag{19}$$

and the stiffer the adhesive layer, i.e., higher modulus or thinner layer, the higher the fracture strength. As a matter of fact, directly from eq. (16) one can find that

$$\left.\frac{\Delta p_{cr}}{p_{cr}}\right|_{\mu \text{ large}} \approx -\frac{1}{2\mu}\frac{\Delta k}{k}. \tag{20}$$

DISCUSSION

There are several points to be emphasized. First, the energy calculations have been simplified by using linear elastic beam theory; large deflections are not admissible in this solution. Second, a Winkler-type elastic foundation, neglecting shear deformation and shear strain energy, has been used. Third, and most important, separation along the line between the plate and adhesive layer has been assumed. It is known that in many cases the fracture progresses *into* either the adhesive or parent material, in which case an alternative calculation is required. Furthermore, in the case that the adhesive layer thickness vanishes, $k = \infty$, a uniform solution results as far as the continuum mechanics solution is concerned, i.e., eq. (14) with $\mu = \infty$. It is not clear from the microscopic point of view, however, that the adhesive fracture energy γ_a is the same for separation of the plate from the rigid base ($\mu = \infty$) as for separation of the plate from the adhesive layer ($\mu \leftarrow \infty$); indeed it is probably not. Finally, it appears that the primary engineering design quantity is neither the layer modulus E' or the layer thickness h' separately but, from eq. (20), the effective spring constant, or foundation modulus, $k \sim (E'/h')$, of the layer. The relative change of the fracture stress with spring constant is inversely proportional quantitatively to the dimensionless wave length $\mu = \lambda a$.

Nevertheless, it is believed that the nature of the results reported herein, and in the process of being extended to circular blister-type plates,[8] is sufficiently quantitative to assess with fair accuracy the importance and tradeoff between modulus and thickness of a vanishingly thin adhesive interlayer, having as its upper limit in strength a value determined by the characteristic specific adhesive fracture energy of the mating surfaces.

It is hoped that these quantitative approximations for adhesive fracture strength will stimulate polymer scientists to describe, at least on a phenomenologic basis, those chemical structure parameters which affect the adhesive surface energy. It would then permit the same type of association to be established as has been demonstrated between the fracture stress and microstructural factors affecting the modulus.[8] In this way a complete link can be made between the macroscopic failure threshold and the chemical constitution of the microstructure, and ultimately improved adhesive strength.

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References

1. H. Dannenberg, *J. Appl. Polym. Sci.*, **5**, 125 (1961).
2. B. M. Malyshev and R. L. Salganik, *Int. J. Fracture Mechanics*, **1**, 114 (June 1965).
3. M. L. Williams, *J. Appl. Polym. Sci.*, **13**, 29 (1969).
4. F. H. Selto and W. B. Jones, *Adhesive Fracture Using Large Plate Deflections*, UTEC ME 69-067, University of Utah, 1969.
5. E. Winkler, *Die Lehre von der Elastizität und Festigheit*, Prague, 1867, p. 182.
6. K. S. Pister and M. L. Williams, Proc. American Soc. of Civil Engineers, October 1960, pp. 992–1005.
7. I. S. Sokolnikoff, *Mathematical Theory of Elasticity*, McGraw-Hill, New York, 1956.
8. J. D. Burton, W. B. Jones, and M. L. Williams, "Theoretical and Experimental Treatment of Fracture in an Adhesive Interlayer," University of Utah, UTEC DO 69-143, December 1969.
9. M. L. Williams and F. N. Kelley, *Proceedings of the Fifth International Congress on Rheology*, **3** (1970).

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